Letters

Comments on "Radiation from Curved Dielectric Slabs and Fibers"

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In the above paper,[1] Lewin gave an expression for the bending loss of a dielectric fiber with circular cross section. I have shown [1] that the bending loss of arbitrary optical guides can be obtained in a simple manner when the field of the straight guide is known. The purpose of this letter is to show that the results of the two theories are in agreement, except for a factor of 2.

In this comparison only the fundamental scalar field $\psi_{00} \equiv \mathrm{HE}_{11}$ is considered.[2] It is assumed that the field in the bent fiber is similar to that of the straight fiber. Because a bend with radius of curvature ρ is equivalent to a linear gradient of refractive index $1/\rho$, it is not difficult to show that the preceding condition holds when $\rho \gg a^3/\lambda^2$, where a denotes the fiber radius and λ the wavelength. The free wavenumber in the fiber material is denoted $k \equiv (\omega/c)n$, and that in the surrounding medium, or cladding, is denoted $k_s \equiv \omega/c$. The elastooptic coefficient of the fiber material is assumed to be zero.

The bending loss is given in [footnote 1, eqs. (55) and (56)]. Let us simplify Lewin's result by assuming that the normalized frequency $F \equiv (k^2 - k_s^2)^{1/2}a$ of the fiber (sometimes denoted V) is large, e.g., $F \gtrsim 3$. In that case, the modified Bessel functions $K_1(y_0)$ in [footnote 1, eq. (55)] can be replaced by $(\pi/2y_0)^{1/2} \times \exp(-y_0)$, where $y_0 \equiv (k_z^2 - k_s^2)^{1/2}a$, and k_z is the axial wavenumber of the straight fiber, $k_z \approx k$. The quantity denoted x_0 in footnote 1 is the first zero of $J_0(x)$, $x_0 \approx 2.4,\cdots$, in the present approximation, and $\epsilon_n = \epsilon_1 = 2$. Thus the bending loss can be rewritten

$$k_{zi} = (x_0^2/\pi^{1/2})(1/F^2a)(y_0\rho/a)^{-1/2} \exp(2y_0) \exp(-\rho/\rho_0') \quad (1)$$

where ρ denotes the radius of the fiber axis, and the critical radius

$$\rho_0' = \tfrac{3}{2}(k^2a^3/y_0^3). \quad (2)$$

Let us further rearrange this result and introduce the quantity

$$s \equiv (k_\varphi^2 - k_s^2)^{1/2} \quad (3)$$

where k_φ is the azimuthal wavenumber at the outer boundary of the fiber, with radius $\rho + a$. Because of conservation of the angular phase velocity of the wave, $k_z\rho = k_\varphi(\rho + a)$. With the approximation $a \ll \rho$ we can therefore write

$$\exp(-\rho/\rho_0') \approx \exp(-\rho/\rho_0)\exp(-2y_0) \quad (4)$$

where ρ_0 is the critical radius defined in [1]

$$\rho_0 = \tfrac{3}{2}k_s^2/s^3.$$

Thus the term $\exp(2y_0)$ in (1) cancels out. In the term in front of the exponential in (1), it is permissible to replace y_0 by F. The expression of the bending loss (in nepers/unit length) of an oversized round fiber given[1] is therefore approximately

$$k_{zi} = (x_0^2/\pi^{1/2})(1/F^2a)(F\rho/a)^{-1/2}\exp(-\rho/\rho_0). \quad (5)$$

The approach used in [1] consists in evaluating the coupling between the fiber mode and the radiation modes. The result is given in the following with a slightly different notation. Let $\psi_u(x,y)$ denote the unnormalized scalar field of the straight fiber. The normalized field is defined as

$$\psi(y) \equiv \psi_u(0,y)\left[\iint k\psi_u^2(x,y)\,dx\,dy\right]^{-1/2} \quad (6)$$

where ψ has dimension $k^{1/2}$. Next we defined the spectral density of the field distribution along the y axis

$$\hat{\psi}^2(k_y) = (1/2\pi)\left[\int_{-\infty}^{+\infty}\psi(y)\exp(-ik_yy)\,dy\right]^2. \quad (7)$$

The bending loss follows from the general expression [1]

$$k_{zi} = \tfrac{1}{2}s\exp(-\rho/\rho_0)\int_{-\infty}^{+\infty}\hat{\psi}^2(k_y)\exp(-\rho s k_y^2/k_s^2)\,dk_y. \quad (8)$$

For the fundamental mode of an oversized circular rod (with arbitrary polarization), the normalized field along the y axis shown in Fig. 1 is

$$\psi(y) = x_0(\pi^{1/2}k^{1/2}aF)^{-1}\exp(-Fy^2/2a^2) \quad (9)$$

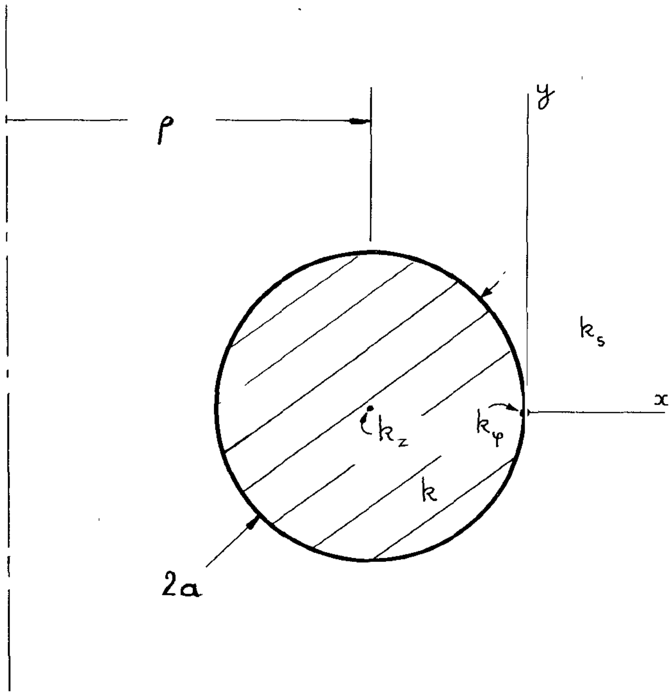

Fig. 1.

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[1] L. Lewin, *IEEE Trans. Microwave Theory Tech.*, vol. MTT-22, pp. 718–727, July 1974.

[2] I have shown in [1, Part II] that Biernson and Kinsley's dispersion formula [2], applicable to step index fibers with small differences of refractive index between core and cladding, follows from the scalar Helmholtz equation. In some recent works, the notation $\mathrm{LP}_{\mu,\alpha+1}$ where LP stands for "linearly polarized," has been used in place of $\psi_{\mu\alpha}$. This alternative notation may be misleading because the polarization of scalar modes is arbitrary. Furthermore, it is not consistent with that commonly used for other types of optical fibers (e.g., square-law graded-index fibers) and hence my preference for the notation $\psi_{\mu\alpha}$, also used in similar problems of quantum mechanics.

where, as before, $x_0 \approx 2.4\cdots$ denotes the first zero of $J_0(x)$. Thus

$$\hat{\psi}^2(k_y) = (x_0^2/\pi)k^{-1}F^{-3}\exp(-k_y^2a^2/F). \quad (10)$$

If we substitute (10) in (8), the bending loss is obtained [1]

$$k_{zi} = (x_0^2/2\pi^{1/2})(1/F^2a)(F\rho/a + k_s^2a^2/F)^{-1/2}\exp(-\rho/\rho_0). \quad (11)$$

This result is just half that in (5) because, for large values of ρ, the second term in the third parenthesis in (11) can be neglected. Snyder and White [3] found a result that agrees with (11). An interesting comparison with experimental results has been recently reported in [4], which, however, is not directly applicable to the present discussion.

In conclusion, the results [1][1] coincide, except for a factor of 2. Lewin's method is, in principle, more rigorous than the one that I proposed. The latter, however, is applicable to more complicated structures. For instance, it is applicable to slab loaded fibers [1].

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[3] A. Snyder and I. White, to be published.

[4] E.-G. Neumann and H.-D. Rudolph, "Radiation from bends in dielectric rod transmission lines," *IEEE Trans. Microwave Theory Tech.* (*Special Issue on Integrated Optics and Optical Waveguides*), vol. MTT-23, pp. 142–149, Jan. 1975.